Imperial Prize and Japan Academy Prize to:

Shihoko Ishii
Emeritus Professor, The University of Tokyo
Emeritus Professor, Tokyo Institute of Technology

for "Multilateral Studies on Singularities"

Outline of the work:

Dr. Shihoko Ishii is one of the international leaders in the study of singularities of algebraic geometry. Singularities appear in all aspects of geometry and analysis, embody their characteristic features, and create various difficulties. One method for overcoming them is the resolution of singularities achieved by Dr. Heisuke Hironaka. There is another standpoint, i.e., treating singularities and their related phenomena as the objects of study with rich content, understanding their natures and structures and classifying "good" singularities if possible. It is essential to allow certain "good" singularities in the framework of the higher dimensional birational algebraic geometry (minimal model theory), which has greatly been developed since the end of the last century. Thus, the study of singularities is considered progressively important, and Dr. Ishii's achievements are remarkable in this stream of research. Her research covers various aspects of singularities including the theory of isolated normal singularities. We will describe three main accomplishments and their influences.

1. Characterization of Du Bois singularities ([3], 1985)

She proved that the following three groups of isolated Gorenstein singularities are equal: (a) the Du Bois singularities described by the Hodge Theory, (b) log canonical singularities in the minimal model theory; and (c) rational or purely elliptic singularities in the classical terminology of singularities. This result attracted significant attention because it proved seemingly different characterizations from different viewpoints to be equivalent, and later it was generalized to the case of nonisolated singularities.

2. Contribution to the Nash Problem ([16], 2003, joint work with J. Kollár)

The arc space of a variety X (the set of all infinitesimal curves on X) is a notion introduced by J. F. Nash, who aimed at considering a natural resolution of singularities. The set of all infinitesimal curves on X passing through some singular point of X consists of a finite number of components (Nash components). Nash (1968) proposed a problem asking whether there is a one-to-one correspondence between the set of the Nash components and that of the so-called essential divisors in Hironaka's resolution of singularities of X. The study [16] constructed a counterexample to the Nash Problem in every four or higher dimension while settling it affirmatively for toric varieties in all dimensions. Having settled a problem negatively that had been open for more than 30 years, their work was a big surprise to the researchers in algebraic geometry and topology.

3. Structure of the arc space of toric varieties ([17], 2004)

It is nearly impossible to concretely describe the arc space of a general algebraic variety. However, in the case of a toric variety X with the action of an algebraic torus T, the quotient space of the arc space of X by the action of the arc space of T can be completely described in terms of toric variety X. This result [17] became a fundamental principle in studying arc and jet spaces and influenced many researchers. For instance, it was used in the theory of horospherical varieties by V. Batyrev–A. Moreau (2013).

In addition to the above three, Dr. Ishii's achievements include important results: the upper semi-continuity of plurigenera of isolated singularities (1986), the injectivity property of the natural functor attaching the morphism of associated arc spaces to that of varieties ([23], 2010, joint work with J. Winkelmann), and the arc space-theoretic description of the minimal log discrepancy, which is a birational invariant of a singularity (2017).

One feature of Dr. Ishii's research is the unpredictability in a good sense. It is demonstrated by the counterexample to the Nash Problem and the discovery of the hidden relationships between various notions (such as Hodge theory and birational geometry, arc spaces and birational geometry). Dr. Ishii's new and original discoveries have affected algebraic geometry and wide areas, such as commutative algebra and topology. She is open to joint research and has many and diverse collaborators.

As stated above, Dr. Ishii's deep and wide academic achievements on singularities and arc spaces deserve the Japan Academy Prize.

List of Main Publications

Papers

[1] Moduli space of polarized del Pezzo surfaces and its compactifications, Tokyo J. Math. 5, (1982) 289–297.
[2] A characterization of hyperplane cuts of smooth complete intersections, Proc. Japan Acad. Ser. A Math. Sci. 58, (1982) 309–311.
[3] On isolated Gorenstein singularities, Math. Ann. 270, (1985) 541–554.
[4] Small deformations of normal singularities, Math. Ann. 275, (1986) 139–148; erratum: Math. Ann. 277, (1987) 351.
[5] Du Bois singularities on a normal surface, Adv. Stud. Pure Math. 8, (1987) 153–163.
[6] Isolated **Q**-Gorenstein singularities of dimension three, Adv. Stud. Pure Math. 8, (1987) 165–198.
[7] Two dimensional singularities with bounded pluri-genera are **Q**-Gorenstein singularities, Contemp. Math. 90, (1989) 135–145.
[8] The asymptotic behavior of pluri-genera for a normal isolated singularity, Math. Ann. 286,

(1990) 803–812.
[9] Quasi-Gorenstein Fano 3-folds with the isolated non-rational loci, Compositio Math. 77, (1991) 335–341.
[10] Simultaneous canonical modifications of deformations of isolated singularities, Algebraic Geometry and Analytic Geometry, Proceeding of the Satellite Conference of ICM 90, Springer Lecture Note, (1991) 81–100.
[11] (with Ki. Watanabe) A geometric characterization of a simple *K*3-singularity, Tohoku Math. J. 44, (1992) 19–24.
[12] The canonical modifications by weighted blow-ups, J. Algebraic Geom. 5, (1996) 783–799.
[13] The quotient of log-canonical singularities by finite groups, Adv. Stud. Pure Math. 29, (2000) 135–161.
[14] (with M. Tomari) Hypersurface non-rational singularities which look canonical from their Newton boundaries, Math. Z. 237, (2001) 125–147.
[15] (with Y. Prokhorov) Hypersurface exceptional singularities, Internat. J. Math. 12, (2001) 661–687.
[16] (with J. Kollár) The Nash problem on arc families of singularities, Duke Math. J. 120, (2003) 601–620.
[17] The arc space of a toric variety, J. Algebra 278, (2004) 666–683.
[18] Arcs, valuations and the Nash maps, J. Reine Angew. Math. 588, (2005) 71–92.
[19] The local Nash problem on arc families of singularities, Ann. Inst. Fourier (Grenoble) 56, (2006) 1207–1224.
[20] Maximal divisorial sets in arc spaces, Adv. Stud. Pure Math. 50, (2008) 237–249.
[21] (with T. De Fernex and L. Ein) Divisorial valuations via arcs, Publ. RIMS 44, (2008) 425–448.
[22] Smoothness and jet schemes, Adv. Stud. Pure Math. 56, (2009) 187–199.
[23] (with J. Winkelmann) Isomorphisms of jet schemes, C. R. Math. Rep. Acad. Sci. Canada 32, (2010) 19–23.
[24] Mather discrepancy and the arc spaces, Ann. Inst. Fourier (Grenoble) 63, (2013) 89–111.
[25] (with A. Reguera) Singularities with the highest Mather minimal log discrepancy, Math. Z. 275, (2013) 1255–1274.
[26] (with L. Ein) Singularities with respect to the Mather-Jacobian discrepancies, Publ. MSRI 68, Vol. II, (2015) 125–168.
[27] (with L. Ein and M. Mustaţă) Multiplier ideals via Mather discrepancy, Adv. Stud. Pure Math. 70, (2016) 9–28.
[28] (with A. Reguera) Singularities in arbitrary characteristic via jet schemes, Adv. Lect. Math. 39, (2017) 419–449.

Book

[1] S. Ishii: Introduction to Singularities, Springer, (2014) 223 pp., 2nd version (2018) 236 pp.