

***Imperial Prize and Japan Academy Prize to:***

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for “Study of Elementary Particle Theory based  
 on Methods of Mathematical Physics”

***Outline of the work:***

Dr. Eguchi may be considered a leading scholar in advancing a new generation of theoretical physicists. Over the past 40 years, he has conducted a deep study of mathematical problems in various areas of particle physics, e.g., gravitation, gauge and string theories, and made significant contributions. This work led to the forming of a new relationship between physics and mathematics and has contributed greatly to the development of both. Dr. Eguchi has also fostered a number of bright young physicists who are now playing important roles in the international community.

**ALE Space**

One of Dr. Eguchi’s most significant results was the discovery of a solution to Einstein equation in the Euclidean region, termed Eguchi-Hanson space [1,2,3]. The Eguchi-Hanson space was initially called a gravitational instanton, but is now also known as an ALE (asymptotically locally Euclidean) space following its generalization by Gibbons and Hawking. ALE spaces are obtained by resolving the singularity at their origin, which appears when a complex 2-dimensional space is divided by some discrete groups. An ALE space contains 2-dimensional spheres whose radii vanish as the parameters of the space are varied. These spheres are called vanishing cycles.

String theory is defined in 10-dimensional space-time and the extra 6 dimensions form a so-called Calabi-Yau manifold. There are various of Calabi-Yau manifolds, but that where the manifold contains ALE spaces is particularly important. This is because recent studies in string theory have shown that in ALE spaces D-branes wrap around the vanishing cycles and generate massless vector particles. These particles behave exactly like gauge fields. Thus in the Eguchi-Hanson space,  $SU(2)$  gauge symmetry is generated while  $SU(N)$  symmetry is generated in general ALE spaces. Accordingly, ALE spaces play a fundamental role in the physics of string theory. Reference [3] is a review of the methods used in of differential geometry, instantons and ALE spaces. They were extensively used by particle physics students during 1980’s and 1990’s.

**Large-N reduction**

In gauge theories like QCD, it had been suggested that some simplification takes place in the limit of a large gauge group  $SU(N)$  with  $N \rightarrow \infty$ . Eguchi, together with H. Kawai, showed that in fact a major simplification takes place in the large-N limit and that gauge theory can be reduced to a simple matrix model [4]. This method is called large-N reduction. The original model of large-N reduction had some difficulty in phase transition when the theory is varied from strong coupling to weak coupling regions. To avoid this difficulty, quenched and twisted versions of the model were proposed. Reduction of field theory to matrix integrals has been rediscovered

repeatedly since the original proposal and is one of the most powerful methods of studying quantum field theory.

### Virasoro Conjecture

Dr. Eguchi has also produced important results on the foundation of conformal field theory together with H. Ooguri [5] and on topological field theory together with S.-K. Yang [6,7,8]. Also, his proposal on Virasoro conjecture in quantum cohomology theory together with K. Hori and C.-S. Xiong [9] had an impact in the area of mathematics related to string theory. Quantum cohomology describes how classical geometry is affected by quantum effects in string theory and Virasoro conjecture asserts that the exact instanton amplitude is annihilated by an infinite series of differential operators which form a Virasoro algebra. In some important cases, Virasoro conjecture has been proven by mathematicians.

### References

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