

Japan Academy Prize to:

Shigeo KUSUOKA
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for “Study of Stochastic Analysis and Mathematical Finance”



Outline of the work:

Stochastic analysis was founded by Dr. Kiyosi Itô, and developed by researchers such as Dr. Shinzo Watanabe. Dr. Shigeo Kusuoka expanded the area significantly by developing methods of infinite dimensional analysis, and by giving profound applications in mathematical finance.

In 1976, Dr. P. G. Malliavin introduced analysis on path spaces, which is now called the Malliavin calculus. This brought a completely new stream to stochastic analysis as infinite dimensional analysis. In his joint work with Dr. D. W. Stroock, Dr. Kusuoka developed and expanded the Malliavin calculus. As an application, they obtained deep results on the hypoellipticity of second order differential operators, where, briefly, the basic question of hypoellipticity is described as follows: suppose that the image of a function by a second order differential operator is smooth, then is it the case that the original function is smooth or not? This is an old problem, and it was already known that the answer is positive if the operator enjoys appropriate smoothness and ellipticity conditions. Dr. Kusuoka weakened these conditions significantly, and resolved the problem affirmatively. This result, based on stochastic analysis, was the leading result at the time, and it had a strong impact on broad areas of mathematics, including analysis and differential equations.

Dr. Kusuoka originated the partial Malliavin calculus, which involves applying the Malliavin calculus to only some of the variables, and applied it to nonlinear filtering problems. He also gave a framework of Riemannian geometry on infinite dimensional spaces by developing a theory of differential forms and localizations on path spaces. His work raises the prospect of developing infinite dimensional de Rham-Hodge-Kodaira theory in a manner that is consistent with stochastic analysis. In his early work, Dr. Kusuoka established fundamental theory on nonlinear transformations on path spaces. In particular, the density formula of the measure appearing as a result of such a transform is called the Ramer-Kusuoka formula and it has many applications.

Dr. Kusuoka made significant achievements concerning fundamental solutions of evolution equations as well. Jointly with Dr. E. A. Carlen and Dr. D. W. Stroock, he proved the equivalence of the upper bound of the fundamental solution for the evolution equation and the Nash inequality, which relates the Dirichlet form and norms of functions. This equivalence implies the stability of upper bounds of the fundamental solutions under perturbations. This equivalence turned out to be extremely useful, and many applications have been made since. It is a milestone in the area of evolution equations.

By applying stochastic analysis, Dr. Kusuoka also produced brilliant results in the area of mathematical finance. Among them it is noteworthy that he gave a systematic way to approximate the expectation of diffusion processes by using the Malliavin calculus and the theory of Lie algebras (this procedure is now called Kusuoka approximation or K-approximation). As a consequence, it is possible to guarantee accuracy in

the high-speed computation of option prices. Furthermore, Dr. Kusuoka proved rigorously that the so-called Ninomiya-Victoir scheme gives a good approximation for option prices, even if the payment function is not smooth.

Dr. Kusuoka discussed financial models of default risks from the viewpoint of stochastic analysis, and showed that a formula of default risk models that had been believed to be true is false in general, which surprised specialists in the area. Concerning the quantification of credit risks, he introduced a notion of probabilistic invariance in the coherent risk measure and characterized invariance in a mathematical manner. This is an important pioneering work in the area. In summary, as we have described, Dr. Kusuoka showed that the infinite dimensional stochastic analysis that he had developed with his collaborators is effective in mathematical finance and related fields.

Dr. Kusuoka undertook pioneering work on stochastic processes on fractals. In the late 1980s, he constructed Brownian motion on the Sierpinski gasket, which is a typical example of a fractal space. This was the first time that Brownian motion had been constructed on a fractal in a mathematically rigorous manner. He also proved that the behavior of the Brownian motion is anomalous in the sense that the diffusive speed is much slower than Brownian motion in Euclidean space. His work is a foundation of the area that developed rapidly later on.

A series of works by Dr. Kusuoka on refinements of the theory of large deviations originally founded by Dr. M. D. Donsker and Dr. S. R. S. Varadhan are profound in the sense that his analysis in infinite dimensions provides detailed information close to that in finite dimensions. Other works of Dr. Kusuoka include, for instance, research on deriving Brownian motion from a system of particles obeying classical mechanics, research on quantum field theory, and research on polymer measures. His research fields are extremely broad.

As we have discussed, Dr. Kusuoka has made significant contributions to the development of stochastic analysis and its applications to mathematical finance.

List of Main Publications

- [1] S. Kusuoka and Y. Morimoto. Least square regression methods for Bermudan derivatives and systems of functions. *Adv. Math. Econ.* **19** (2015), 57–89.
- [2] S. Kusuoka and Y. Morimoto. Stochastic mesh methods for Hörmander type diffusion processes. *Adv. Math. Econ.* **18** (2014), 61–99.
- [3] S. Kusuoka. Gaussian K-scheme: justification for KLVN method. *Adv. Math. Econ.* **17** (2013), 71–120.
- [4] S. Kusuoka. A remark on Malliavin calculus: uniform estimates and localization. *J. Math. Sci. Univ. Tokyo* **19** (2012), no. 4, 533–558.
- [5] S. Kusuoka and S. Liang. A classical mechanical model of Brownian motion with plural particles. *Rev. Math. Phys.* **22** (2010), no. 7, 733–838.
- [6] S. Kusuoka, K. Kuwada and Y. Tamura. Large deviation for stochastic line integrals as L^p -currents. *Probab. Theory Related Fields* **147** (2010), no. 3–4, 649–674.
- [7] S. Kusuoka and Y. Osajima. A remark on the asymptotic expansion of density function of Wiener functionals. *J. Funct. Anal.* **255** (2008), no. 9, 2545–2562.
- [8] S. Kusuoka and S. Ninomiya. A new simulation method of diffusion processes applied to finance. *Stochastic processes and applications to mathematical finance (Kusatsu, 2003)*, 233–253, World Sci. Publ., River Edge, NJ, 2004.

- [9] S. Kusuoka. Approximation of expectation of diffusion processes based on Lie algebra and Malliavin calculus. *Adv. Math. Econ.* **6** (2004), 69–83.
- [10] S. Kusuoka. Malliavin calculus revisited. *J. Math. Sci. Univ. Tokyo* **10** (2003), no. 2, 261–277.
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- [16] S. Kusuoka. Limit theorem on option replication cost with transaction costs. *Ann. Appl. Probab.* **5** (1995), no. 1, 198–221.
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